SUPPLEMENTARY MIDTERM 2019: COMP. SCIENCE. II (B.MATH. 2ND YEAR)

- *This is a pen-and-paper exam, use of computers/calculators is not allowed.*
- *This is a closed book exam.*
- *The question paper is of* 50 *points. All questions are compulsory.*

Problem 1 (1 points each)**.** Answer the following questions in at most one sentence each.

(1) Write down the output of following statement in `SageMath`: `a = 2^100000+1; b=a+1; print (b-a)`.
(2) Write down the output of following command in `SageMath`: `print [1,2]*2`.
(3) Why is recursion $x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}}$ is preferable over $x_{n+1} = \frac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n)-f(x_{n-1})}$ in the Secant method?
(4) Let A be any matrix. Do A^2 and A necessarily have the same rank? (give a counter example if false).
(5) Why does implementation of real numbers in `SageMath` will roundoff 3.5 (11.1 in binary) to the higher number 4 (100 in binary) while it will round off 2.5 (10.1 in binary) to the lower number 2 (10 in binary) when working with two bits of precision?

Problem 2 (5 points)**.** Write down a `Python` function which takes one parameter n as input and returns n-th Fibonacci number as output.

Problem 3 (10 marks)**.** Let $f(x) = \ln(1-x)$ (here ln denotes natural logarithm, that is log to base e). Write down it's Taylor series expansion around $x = 0$. Estimate till how many terms should we sum the Taylor series to calculate $f(1/2)$ to an accuracy of 53 bits.

Problem 4 (10 marks)**.** Find a bound for the number of iterations of Newton-Raphson method needed to achieve an approximation with accuracy 2^{-8} to the solution of $x^3 + x - 3 = 0$ lying in the interval $[1, 3]$ (assume a reasonable initial guess for which the method converges). What is the numbers of steps, were we to work with bisection method instead?

Problem 5 (10 marks)**.** Let x_n be the n-th guess of the Newton-Raphson method, converging to a root r of a function f. If e_n denotes the error $|x_n - r|$, show that under reasonable assumptions on f the error e_{n+1} is bounded by $C \cdot e_n^2$. State the corresponding assumptions on f clearly and compute the constant C explicitly.

Problem 6 (10 marks)**.** Compute LU-decomposition of the following matrix (that is a permutation matrix P, a lower triangular matrix L (with ones on the diagonal), and an upper triangular matrix U s.t. $PA = LU$):

$$A = \begin{pmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{pmatrix}$$

−THE END−